INVENTED ALGORITHMS: TEACHERS FACE THE CHALLENGE

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This paper explores teachers' responses when implementing a teaching approach that encouraged children to develop and use invented algorithms for computation. The children demonstrated number sense and used methods that predominantly reflected left-to-right computational processes. While acknowledging the children's success and their ability to explain and justify the methods used, the teachers faced a number of challenges. They confronted previously held beliefs, perceived curriculum requirements and social expectations regarding the place of standard algorithms for computation.

Background to the Study

The debate surrounding the use of standard algorithms for numerical computation is not new. Ginsburg (1977) suggested that children "fail to understand the necessity or rationale for such methods ... The result is not only a bizarre written arithmetic, but a gap between it and children's informal knowledge" (p125). In contrast to standard formats children's invented algorithms often reflect a left-to-right mental process. It is argued that children resolve the apparent conflict between their own mental or written strategies and the standard algorithm by giving up their own thinking (Kamii & Dominick, 1998). More recently greater emphasis has been placed on children's development of number sense and mental computation skills rather than mere fluency in the use of algorithms. It has been well documented that children develop a meaningful understanding of numbers if given the opportunity to use their own procedures (e.g. Carpenter, Franke, Jacobs, Fennema & Empsom, 1998; Kamii, 1989). It is believed children's number sense allows their understandings to be used in flexible ways (McIntosh, Reys, Reys, Bana & Farrell, 1997) and is a factor in allowing them to depart from traditional or taught algorithmic methods (Bobis, 1996).

Mathematics in the New Zealand Curriculum (Ministry of Education, 1992) suggests children develop flexibility and creativity in applying mathematical ideas and makes no reference to teaching the traditional algorithm. Nevertheless the teaching of traditional algorithmic methods for multidigit computation remains common practice in New Zealand classrooms. Rather than debating the place of children's invented algorithms as opposed to traditional taught algorithms discussion in teaching circles has tended to focus on which methods to teach (eg decomposition or renaming as opposed to equal additions or "borrow and pay back" methods of subtraction). The use of place value blocks to model the algorithmic processes has also been a major focus in New Zealand classrooms. Teaching approaches advocating the use of place value blocks were tightly structured. The four operations were taught in sequence with expected mastery of multidigit addition before the introduction of multidigit subtraction. For each operation children would be presented with examples requiring no regrouping of place value blocks before being introduced to examples requiring regrouping (eg 10 ones renamed as 1 ten). The use of structured equipment was aimed to assist children to develop a mental image of the computational process however, rather than reflecting the mental strategies used by the children the approaches were teacher directed and designed to lead children to the standard algorithms for computation.

Literature shows that while children have the ability to develop effective strategies for solving computational problems, most teachers continue to expect the use of standard algorithms (Peters, 1997). The work of authors recommending emphasis on number sense and invented algorithms is well known by mathematics educators in New Zealand. However, while many teachers have introduced into their mathematics programmes activities that encourage mental computation and estimation, few have extended this approach to include children using their own invented algorithms. If more emphasis is to be placed on identifying and developing children's legitimate strategies (Cooper, Heirdsfield & Irons, 1996) what challenges do teachers need to be prepared to face?

The Research Study

This study set out to explore the implications for teachers when implementing a change in their mathematics instruction for multidigit computation. It follows the experiences of two teachers in an Auckland primary school as they encouraged their children, aged 7-9 years, to develop and implement their own mental strategies and algorithms. It views not only the classroom as a site of social construction of concepts, but also the teachers' wider community as a site in which beliefs and pedagogical practice are constructed.

The school was approached with the offer of professional development in which invented algorithms would be demonstrated as a way of encouraging number sense. Identifying this as in keeping with the school's general policy of developing children as independent thinkers, the principal accepted the offer and two teachers volunteered to participate in subsequent implementation of the approaches in their classrooms. The professional development took place over six months, with three initial meetings followed by a period in which the two teachers were observed in their classrooms.

The methodology of the study was a case study of the two classrooms with the author as participant observer. During a five-week period I joined each class on three or four occasions each week when children were working with a unit of work that focused on multidigit computation. Classes were videotaped and audiotaped and samples of children's work were collected every 3 or 4 school days. While my major focus was to observe the classroom interactions, it was agreed that I would offer support for the children and teachers as required during the ongoing classroom activity. Following each observation session the teacher and I discussed the day's activities and considered any issues that arose. At the conclusion of the five-week observation period each teacher participated in an interview during which they discussed their reactions to the implementation of the teaching approach.

The teachers, Ms N and Mr C, were both experienced primary school teachers. Their approaches to the teaching of multidigit computation had previously focused on work with place value blocks, the development of the standard algorithm and exploration of computational word problems. Neither teacher had experience with children implementing their own invented algorithms. Ms N, who taught a Year 3 class (aged 7-8), had 14 years teaching experience but expressed a lack of confidence in her own abilities in mathematics and a perceived need for support in the teaching of mathematics. Mr C, who taught a Year 3&4 class (aged 7-9) had 20 years teaching experience and was confident in both his own mathematics and his teaching of mathematics.

Results

Ms N introduced her "number unit" by revising addition and subtraction basic facts with the class. She extended this activity by subsequently asking children to record multiple names for numbers (eg 19=10+9, 19=1000-981). She then focused on two-digit place value asking children to identify the tens and ones values (eg 36 is 30 and 6 or 3 tens and 6 ones). Multidigit computation was formally introduced by providing children with addition problems involving values that required no renaming of ones as tens (ie 54+38 would not have been included during this introduction as the ones values when added exceed 10). Children were encouraged to use any methods that resulted in an accurate answer and to share their methods with others in the class. She suggested that children look for ways they could use known information to solve more difficult problems. She focused on how basic facts knowledge could be used to solve problems with larger numbers (eg if 3+4=7, then 30+40=70 and 300+400=700). As children displayed confidence adding two-digit numbers with no renaming Ms N introduced examples where renaming was required (ie the ones values when added equalled more than 10). Toward the end of the five-week unit Ms N introduced subtraction, although she expressed a concern that some children "may not be ready for it".

Although her planning appeared tightly structured and influenced by her earlier experiences using place value blocks, throughout the five-week unit Ms N encouraged children to express their own ideas and to share these with others. Opportunities for discussion were introduced into each day's activity and all children were encouraged to participate in an environment where all contributions were openly valued and received with interest. Recordings were presented in formats that reflected the spoken descriptions given by the children and although two children had home experience with standard algorithms these were not formally introduced in this unit.

Acknowledging the age and ability range within his class, Mr C decided to allow flexibility for some children to extend beyond addition and subtraction during the five-week unit. He began the unit by assessing children's skills in multidigit addition and subtraction. Children were set two word problems to work on as homework. It was emphasised that the problems, requiring addition of 53 and 39 and subtraction of 27 from 62, were to be solved using any method and children were asked to use words or numbers to write down how they "got the answer". Within classroom activity Mr C focused on similar introductory activities as used by Ms N however these activities included greater emphasis on multiplication and division as well as addition and subtraction. He focused on the inverse nature of multiplication and division and explored strategies for mental computation such as the use of doubles and derived facts. Multidigit computation problems were posed in contexts (as word problems) by the teacher or selected by the children as a Lucky Dip from which they chose two or more counters each with 1-digit, 2-digit or 3-digit numbers that could be added, subtracted, multiplied or divided. This latter activity allowed children to work at their own level but Mr C also monitored children's choices.

Children were encouraged to use mental methods as well as record the way in which they worked. As in Ms N's classroom, children were encouraged to share their ideas and extensive opportunities were provided for discussion each day. The phrase, "Does anyone have a different way of working this out?" was used in most discussions and children were openly encouraged to look for alternative methods to try. While Mr C was openly committed to children devising and using their own strategies he also perceived greater pressure from parents and future teachers to introduce the standard algorithm. At the midpoint of the unit he introduced the children to the algorithm by demonstrating on the whiteboard the way he would record the problem. Although he emphasised that this was only one way of recording there was a marked change in the recording used by the children after this demonstration.

Children in both classrooms demonstrated a range of strategies for computation, for example: counting on, counting back, addition or subtraction by place values, use of known facts and use of compensation. Children's written work samples were collected at seven regular intervals throughout the unit and were analysed in terms of presentation (whether the children recorded their working using horizontal or vertical working formats) and process direction (whether the children started with the larger values or the units when calculating). Samples from the first five collection points in Ms N's classroom (Class 1) reflected addition and the final two sets of samples showed subtraction examples. The first two sets of samples collected from Mr C's classroom (Class 2) were the addition and subtraction homework examples completed by the children. Subsequent sample sets included addition, subtraction and multiplication examples. Table 1 indicates the way in which children presented their work at each sample collection point and Table 2 shows the process direction used by the children.

Most children in Ms N's class used horizontal working formats throughout the five-week unit. Those children using vertical formats identified they had received instruction from the home in the use of these methods however it was noted that all of these children used horizontal formats on other occasions. Collection point 6 reflected samples collected when subtraction was first introduced in this classroom and several children showed no working.

Table 1
Percentage of children at each sample collection point using horizontal and vertical formats for calculating. Class 1: n=22, Class 2: n=23

Data Collection Point	1	2	3	4	5	6	7
Class 1 Horizontal	91%	100%	100%	95%	95%	77%	95%
Class 1 Vertical	9%	0%	0%	0%	0%	0%	4%
Class 2 Horizontal	39%	70%	78%	43%	43%	56%	56%
Class 2 Vertical	57%	30%	17%	57%	57%	43%	43%

The children in Mr C's class demonstrated greater use of vertical formats. Some children had had prior experience with the standard algorithm in their previous year of schooling. It was also noted on some work samples that children had received assistance from parents or siblings when completing their homework task. As the children explored new ideas and were encouraged to use their own formats the number of vertical representations decreased however there was a marked change following Mr C's demonstration of his vertical format, the standard algorithm, prior to collection point 4.

Table 2
Percentage of children at each sample collection point calculating larger place values before units (left-to-right) and units before larger place values (right-to-left). Class 1: n=22, Class 2: n=23

Data Collection Point	1	2	3	4	5	6	7
Class 1 Left-to-right	59%	82%	82%	73%	77%	27%	41%
Class 1 Right-to-left	14%	4%	4%	0%	0%	0%	4%
Class 2 Left-to-right	39%	22%	35%	4%	0%	0%	0%
Class 2 Right-to-left	26%	22%	0%	4%	13%	17%	26%

The process direction used by children in their recordings was often difficult to establish from the written work samples. This was most evident in Mr C's class with children more frequently used the vertical formats. Where problems required no renaming (eg 10 ones as 1 ten) it was unlikely that there was any indication whether the children had started with the units, tens or larger values when calculating. The predominance of a left-to-right process in Ms N's class reflected the frequent use of horizontal recordings by the children. Problems were approached as they would be read (ie left-to-right) and recording clearly indicated the children calculating tens values before units (eg $30 + 50 = 80, 5 + 7 = 12$ so $35 + 57 = 92$).

Discussion

While acknowledging the children's success and their ability to explain and justify the methods used, the teachers faced a number of challenges within the context of their own beliefs, the classroom programme and school and community expectations.

The teachers identified that children completed fewer examples but spent more time discussing their work with the teacher and others in the class, reflecting greater emphasis on the verbalisation of the processes used. They believed that the frequent opportunities for discussion increased children's confidence in presenting their ideas to others. Mr C commented that children were *"thinking deeper"* and that they were *"freed to think for themselves, to challenge themselves, to build independence"* and to no longer be *"bounded by the task"*. Mr C and Ms N both acknowledged that many children exhibited skill levels beyond their expectations and noted that in a traditional classroom programme some of these children would have been limited to practising skills they had already mastered.

Both the teachers identified their role as encouraging children to put forward their ideas and valuing these contributions. Ms N openly expressed to the children the dilemma she faced when introducing something new to the class. She said, *"Part of me wants to get out there and show you how to work it out but I'm going to let you have a think about it first. Then you (emphasised on tape) can decide how best to work it out to suit your way of thinking."* She recognised such statements as reflecting her perceived lack of knowledge in mathematics. When reflecting on her development of the number unit with the class Ms N identified that she had imposed limits on the range of problems to be explored in order that she was better able to predict possible difficulties and prepare for these. She also acknowledged that the structure of her teaching, although encouraging children's own methods, was similar to the format of instruction she had used when working with place value blocks, an approach with which she was familiar and had experienced perceived success. During the final interview Ms N indicated an intention to include a wider range of examples at an earlier stage together with opportunities for children to extend their ideas.

Although there was an acceptance that children could develop their own methods for computation, the teachers expected that these methods should also lead to accurate responses. Throughout the five-week unit high levels of accuracy were evident in the activities observed and samples collected. However in each classroom these levels dropped for a short time. Given Ms N's continued emphasis on addition before the introduction of subtraction, the children in her class had strongly developed successful methods (most commonly $46 + 35$ would be solved as $40 + 30 = 70$, $6 + 5 = 11$ so $46 + 35 = 81$) that they then assumed could be applied to subtraction. In attempting to apply these methods, many children made a number of errors (eg $52 - 35$ solved as 50-30=20, 2-5=3 so 52-35=23) however during the next lesson successful strategies were being shared and implemented successfully. In Mr C's class the number of errors increased markedly after the introduction of the standard algorithm as children attempted to apply this method. At this time one child was recorded as saying, *"I know the answer should be ... but I can't get the numbers to work"*. She had successfully calculated the solution mentally but in applying the standard algorithm was achieving an incorrect result.

As pointed out by Yackel, Cobb and Wood (1992), teachers work in a context in which procedures must be institutionally sanctioned. Both teachers highlighted the expectations they perceived from parents and subsequent class level teachers. Parents consider the acquisition of computational skills, along with the knowledge of the tables, as the most important mathematical task of primary schooling (McIntosh, 1998) and demonstration of these skills is seen to be through paper-and-pencil computation using standard working formats. Mr C commented that his introduction of the standard algorithm was in response to a perceived demand from parents that children know these methods. Although the use of invented algorithms had been accepted by the school as appropriate for implementation it was evident that both teachers saw this approach as an interim step and that standard algorithms would still be expected subsequent to children's exploration of their own methods. They expressed concern that future teachers might wonder what they had been teaching all year if the children had no experiences of standard algorithms. They recommended that any school taking on this approach should view it as a school-wide policy whereby all teachers were familiar with expectations and parents could be kept informed. Mr C pointed out, *"If it is going to happen you need a climate that asks people to take the risk of trying something new from the start."*

While accepting the benefits to children of using this approach the teachers perceived expectation of the wider community acted as a constraint in making changes to their teaching. For change to be implemented in the New Zealand setting it appears that the focus must come from the wider educational community and be backed up with support for teachers and schools.

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